

Summary

Wind gusting on the secondary mirror cell and support structure in the CELT will decenter the secondary, causing comatic images. The standard deviation of the decenter can be as much as $\sim 40 \mu\text{m}$ for a mean outside wind speed of 14 ms^{-1} , with a simple 4-leg secondary support, and assuming a dome geometry similar to that for Gemini. The coma due to a $40 \mu\text{m}$ secondary decenter is $\sim 15 \text{ mas}$. At $\lambda = 1 \mu\text{m}$, the diameter of the Airy disc for the CELT is $\sim 8 \text{ mas}$, so aberrations due to wind gusts will degrade the optical performance of the telescope. If the CELT dome is made as small as possible, i.e. $\sim 90 \text{ m}$ in diameter, the standard deviation of the secondary decenter can be as much as $\sim 80 \mu\text{m}$, and the coma $\sim 30 \text{ mas}$. The decenter could be reduced by making a stiffer secondary support, or by actively correcting the position of the secondary at $\sim 1 \text{ Hz}$.

1. Wind model

Wind speed measurements [1] inside the Gemini South dome show $\langle v_p \rangle \sim 0.05 \langle v_{ext} \rangle$, where v_{ext} is the wind speed outside the dome and v_p is the wind speed at the primary. The standard deviation of the wind speed at the primary is $\sigma_{v_p} \sim 0.5 \langle v_p \rangle$. The wind at the secondary is not so well-behaved, and the Gemini measurements show mean wind speeds in the range $\langle v_s \rangle \sim 0.10 \langle v_{ext} \rangle$ to $\langle v_s \rangle \sim 0.35 \langle v_{ext} \rangle$, and standard deviations in the range $\sigma_{v_s} \sim 0.35 \langle v_s \rangle$ to $\sigma_{v_s} \sim 0.95 \langle v_s \rangle$. It is not possible to construct an accurate wind model from these results, but we can make a very simple model in which both the mean wind speed, and the standard deviation of the wind speed, increase away from the center of the dome

$$\langle v(h) \rangle = \alpha \frac{h}{h_s} \langle v_{ext} \rangle \text{ and } \sigma_v(h) = \beta \frac{h}{h_s} \langle v(h) \rangle$$

where h is the distance from the center of the dome, h_s is the distance from the center of the dome to the secondary and α and β are constants. This model underestimates the wind near the primary, but this region is not very important for estimates of the secondary decenter. For Gemini South, $\alpha \sim 0.35$ and $\beta \sim 0.9$ give an upper limit to the wind at the secondary. The wind will be higher for the CELT if the ratio of dome opening to dome diameter is larger than for Gemini, and if the secondary is closer to the dome opening. The Gemini secondary is 3 m inside a 36-m diameter dome with a 10 m opening. If the CELT dome is made as small as possible, the secondary will be $\sim 3 \text{ m}$ inside a $\sim 90\text{-m}$ diameter dome with a $\sim 32.5 \text{ m}$ opening. Assuming that the mean wind speed inside the dome scales with the ratio of dome opening to dome radius [1], the wind speed at the CELT secondary will be about $\frac{32.5/45}{10/18} \times \frac{42/45}{15/18} = 1.46$ higher than at the Gemini secondary, i.e. $\alpha \sim 0.5$. For the following estimates we will consider $\alpha = 0.35$ and 0.5 and $\beta = 0.9$.

The pressure on the secondary mirror cell and support structure is

$$P(h) = \rho v^2(h), \quad \langle P(h) \rangle \sim \rho \alpha^2 \left(\frac{h}{h_s}\right)^2 \langle v_{ext} \rangle^2 \text{ where } \rho \text{ is the density of air.}$$

The standard deviation of the pressure is

$$\sigma_P(h) = 2\rho \langle v(h) \rangle \sigma_v(h) = 2\rho \alpha^2 \beta \langle v_{ext} \rangle^2 \left(\frac{h}{h_s}\right)^3$$

The secondary decenter due to the mean wind pressure can be corrected along with low-order modes in the primary, using the same wavefront measurements. The following discussion applies just to the turbulent part of the decenter.

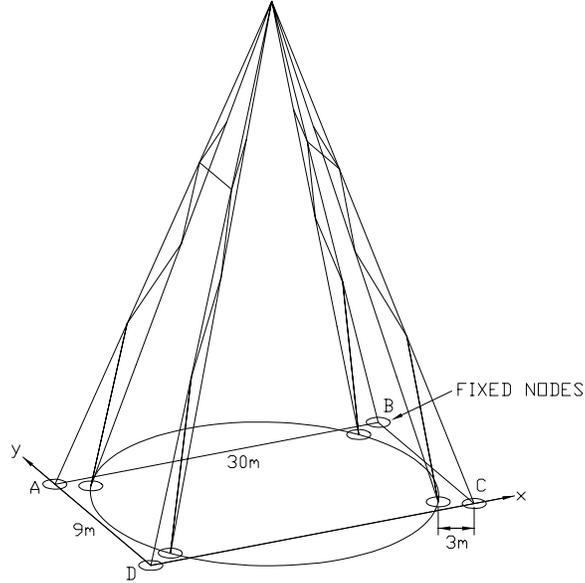


Figure 1: Simple model of the secondary support structure. The elements are steel tubes, 1 m in diameter with a 10 mm wall thickness, modelled as beams. The bottom of each leg is fixed.

2. Secondary support structure

To estimate the stiffness of the secondary support under wind loading, we used the simple model shown in Fig. 1. This is essentially the CELT-5B concept described by Woody [2]. The secondary support used for the CELT will probably be stiffer, so the model overestimates the secondary decenter due to the legs bending. However, the model does not include decenter due to deformations where the legs attach to the rest of the telescope. The stiffness of the structure was estimated by measuring the displacement of the apex, with nodal forces applied to legs A and B in the y direction, with the magnitude of the force varying as $(\frac{h}{h_s})^3$, which is appropriate for the turbulent part of the wind force. With legs made of 1-m diameter, 10-mm wall, steel tubes, the stiffness is $K \sim 10^8 \text{ Nm}^{-1}$ i.e. a total force of 1000 N applied to legs A and B moves the apex $\sim 10 \mu\text{m}$.

The wind cross-section of each leg does not vary substantially along the leg, and to estimate the total force on the leg we assume that the horizontal width w is constant. Each leg in the support structure has 2 long tubes and a tube for the web, so $w \sim 3 \text{ m}$. If the wind gusts are correlated over the length of a leg, the standard deviation of the total force on the leg is

$$\sigma_{F_{leg}} = \int_0^{h_s} w \sigma_P(h) dh = 2w\rho\alpha^2\beta \langle v_{ext} \rangle^2 \int_0^{h_s} (\frac{h}{h_s})^3 dh = \frac{1}{2}wh_s\rho\alpha^2\beta \langle v_{ext} \rangle^2$$

The wind cross-section is roughly 2 legs, so the standard deviation of the displacement at the apex is

$$\sigma_{\delta_{leg}} = \frac{wh_s\rho\alpha^2\beta\langle v_{ext} \rangle^2}{K}$$

To estimate the displacement of the apex due to wind gusting against the secondary mirror cell, we calculated the stiffness of the model structure using a force applied just at the apex. This gives $8.5 \times 10^7 \text{ Nm}^{-1}$, which is close to the stiffness from the distributed force estimate. (This is expected since the model wind force varies as h^3 , so most of the force is on the upper part of the structure.)

The standard deviation of the force on the secondary mirror cell is

$$\sigma_{F_{cell}} = 2A_{cell}\rho\alpha^2\beta\langle v_{ext} \rangle^2 \text{ where } A_{cell} \text{ is the wind cross-section of the cell.}$$

The standard deviation of the displacement is

$$\sigma_{\delta_{cell}} = \frac{2A_{cell}\rho\alpha^2\beta\langle v_{ext} \rangle^2}{K}$$

For $\rho = 1.29 \text{ kg m}^{-3}$, $\alpha = 0.35$, $\beta = 0.9$, $v_{ext} = 14 \text{ ms}^{-1}$ (95th percentile at Gemini South), $h_s = 45 \text{ m}$, $w = 3 \text{ m}$, $A_{cell} = 12 \text{ m}^2$ and $K = 10^8 \text{ Nm}^{-1}$, $\sigma_{\delta_{cell}} = 6.7 \text{ }\mu\text{m}$ and $\sigma_{\delta_{teg}} = 37.6 \text{ }\mu\text{m}$. If $\alpha = 0.5$, $\sigma_{\delta_{cell}} = 13.7 \text{ }\mu\text{m}$ and $\sigma_{\delta_{teg}} = 76.7 \text{ }\mu\text{m}$. These are probably overestimates, because they assume that wind gusts are completely correlated over the structure. If the correlation length is l , and $l \ll h_s$, $\sigma_{\delta_{teg}} \sim \frac{wl\rho\alpha^2\beta\langle v_{ext} \rangle^2}{K}$. If l is of the same order as h_s , $\sigma_{\delta_{teg}}$ is not changed substantially because the model wind force varies as h^3 .

3. Aberrations

The aberrations due to secondary decenter δ are [3]

$$\text{Image motion } IM\delta = \frac{\delta}{f_1} \left(1 - \frac{1}{m}\right)$$

$$\text{Length of tangential comatic image } ATC\delta = \frac{3}{32} \left(\frac{m-1}{F}\right)^3 \frac{\delta}{D_1} \left(k_2 - \frac{m+1}{m-1}\right)$$

$$\text{Diameter of astigmatic blur circle } AAST\delta = \frac{\delta}{D_1} \left(\frac{m-1}{2F}\right)^2 \left[\frac{\delta}{D_1} \frac{(m^2-1)(k_2+1)}{2F(1+\Delta)} + 2\theta \left(1 + \frac{(m-1)(m-\Delta)(k_2+1)}{2m(1+\Delta)}\right) \right]$$

where

$D_1 = 30 \text{ m}$	primary diameter
$f_1 = 45 \text{ m}$	primary focal length
$f = 450 \text{ m}$	final focal length
$k_2 = -1.525$	conic constant of secondary
$m = \frac{f}{f_1} = 10$	magnification by secondary
$e = 15 \text{ m}$	back focal distance
$\Delta = \frac{e}{f_1} = 0.333$	
$\theta = 10^\circ$	angular radius of field

For $v_{ext} = 14 \text{ ms}^{-1}$ and $\alpha = 0.35$, the standard deviation of the decenter is $\sim 40 \text{ }\mu\text{m}$, and the corresponding aberrations are $IM\delta \sim 165 \text{ mas}$, $ATC\delta \sim 15 \text{ mas}$ and $AAST\delta \sim 250 \text{ }\mu\text{as}$. If $\alpha = 0.5$, the standard deviation of the decenter is $\sim 80 \text{ }\mu\text{m}$, and $IM\delta \sim 330 \text{ mas}$, $ATC\delta \sim 30 \text{ mas}$ and $AAST\delta \sim 500 \text{ }\mu\text{as}$. For comparison, the diameter of the Airy disc at $\lambda = 1 \text{ }\mu\text{m}$ is 8.4 mas . The image motion due to wind-induced secondary decenter can easily be corrected by an adaptive optics system, but the corresponding coma cannot. Thus, for an outside wind speed of $\sim 14 \text{ ms}^{-1}$, the image quality at $\lambda = 1 \text{ }\mu\text{m}$ will be wind-limited.

4. Discussion

If the geometry of the CELT dome is similar to that for Gemini, i.e. $\alpha = 0.35$ (which requires a 112-m diameter dome with a 32.5 m opening and the secondary 11 m from the opening), and if the secondary support can be made a factor 3 or 4 stiffer than the simple 4-leg design, i.e. $K \sim 3$ or $4 \times 10^8 \text{ Nm}^{-1}$, wind-induced secondary decenter will not be a problem. If the structure cannot be made stiff enough, or if the CELT dome is only $\sim 90 \text{ m}$ in diameter, we will have to actively correct the secondary decenter. Most of the power in the Gemini wind measurements is at frequencies $\lesssim 1 \text{ Hz}$ [1],

so most of the decenter could be removed by applying corrections at ~ 1 Hz. The first resonance in the CELT secondary support structure will be at a few Hz [2], so correcting the secondary position at 0.5–1 Hz should not excite the structure. Active control will require measurements of the secondary decenter on timescales of a few hundred ms. Ideally, the measurements would come from the wavefront sensor which is measuring the low order modes in the primary, but a mechanical measurement is also possible. We could, for example, measure the distance from the edge of the primary to the edge of the secondary at three or four places. This would be fast, but the measurements would be affected by wind-induced deformations in the primary and secondary mirror supports.

A better estimate of secondary decenter will require a more accurate wind model. Unfortunately, this is not easy and will probably involve a combination of wind tunnel tests, simulations, and further measurements at an existing telescope.

References

- [1] D. MacMartin, “Conclusions from Gemini Wind Data” 2001
- [2] D. Woody, “Short Report on Supporting the CELT Secondary” 2001
- [3] J. Nelson, T. Mast and G. Chanan, “Aberration Correction in a Telescope with a Segmented Primary” SPIE 1114, 241-257, 1989